Interplanetary Cruising

Contents

1. Foreword

The following material is intended to supplement NASA-JSC's High School Aerospace Scholars (HAS) "Liftoff!" instructional reading associated with the existing *Mars on the Mind* Student Lesson #7[1]. On request, the author can supply "Mission" (assignments), "Extended Mission" (optional links and information), and "Quick Quiz" (questions) input associated with this *Interplanetary Cruising* topic.

The author, a retired veteran of 60 Space Shuttle flights at Mission Control's Flight Dynamics Officer (FDO) Console, first served as HAS mentor in association with a June 2008 workshop. On that occasion, he noticed students were generally unfamiliar with the nature and design of spacecraft trajectories facilitating travel between Earth and Mars. Much of the specific trajectory design data appearing herein was developed during that workshop as reference material for students integrating it into Mars mission timeline, spacecraft mass, and cost estimates. The *Interplanetary Cruising* topic is therefore submitted to HAS for student use both before and during Mars mission planning workshops.

2. Introduction

Material in this lesson will help you understand the pedigree and limitations applying to paths (trajectories) followed by spacecraft about the Sun as they "cruise" through interplanetary space between Earth and Mars. Although the most advanced propulsion systems in existence now permit interplanetary travel under continuous thrust, we'll simplify these more exotic trajectories by assuming all your spacecraft's rocket power is delivered as you depart Earth for Mars or vice-versa. In effect, your cruise will be a coast subject only to acceleration from gravity.

As modeled by Johannes Kepler in 1605 to high accuracy, the orbits of Earth and Mars about the Sun are near-circular ellipses with the Sun lying at one focus. This principle is known as Kepler's First Law and is a manifestation of Isaac Newton's Law of Universal Gravitation published in 1687. Except for a few days after departing or before arriving

[1] Note the HAS home page for this lesson is titled "Mars of [sic.] the Mind". The author is citing this title with "on" replacing "of" based on a link to Lesson #7.

at Earth or Mars, your coasting spacecraft's trajectory is only affected to an appreciable extent by solar gravitational acceleration and obeys Kepler's First Law too. Consequently, we'll make the further assumption that interplanetary cruise is only subject to acceleration from the Sun's gravity.

This last assumption lets us invoke a very powerful spacecraft trajectory design technique known as Lambert's Theorem. Published by Johann Lambert in 1761, Lambert's Theorem relates cruise time Δt between Earth and Mars to three geometric quantities. Each of the geometric quantities, defined below, is illustrated in Figure 1 for an example spacecraft trajectory from Earth to Mars[2].

1) $a \equiv$ mean solar distance (semi-major axis) pertaining to the cruise ellipse
2) $r_D + r_A \equiv$ the sum of Sun-centered (heliocentric) distances at departure and arrival in the cruise ellipse
3) $c \equiv$ the distance (chord) between heliocentric departure and arrival positions in the cruise ellipse

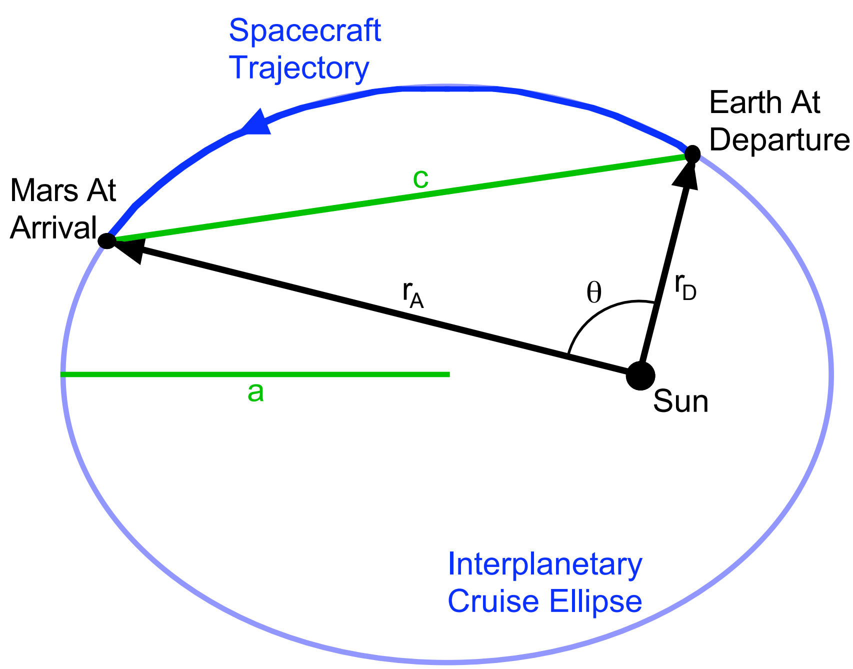

Figure 1: Lambert's Theorem Parameters In An Earth-To-Mars Cruise Ellipse

[2] Figure 1 exaggerates the difference between Earth and Mars distances from the Sun for clarity.

3. Lambert Solutions

Lambert's Theorem is typically applied to solution of a boundary value problem in which departure time t_D and departure planet are specified, together with arrival time t_A and arrival planet. These Lambert boundary conditions (LBCs) are used with well-documented theories of Earth and Mars orbit motion (ephemerides) to generate heliocentric vector positions $\mathbf{r}_D$ and $\mathbf{r}_A$. From these positions, scalars r_D, r_A, and c are readily computed, and $\Delta t = t_A - t_D$. The Lambert equation relating Δt to a, $r_D + r_A$, and c is then iteratively solved for a, defining the interplanetary cruise ellipse.

The spacecraft trajectory illustrated by Figure 1 is known as a "short-way" Lambert solution because its heliocentric arc subtends an angle $\theta < 180°$. A second Lambert solution defined by Figure 1's $\mathbf{r}_D$ and $\mathbf{r}_A$ is the "long-way" trajectory on a different interplanetary cruise ellipse[3] and subtending $360° - \theta > 180°$. In practice, we'll therefore supplement LBCs with a short-way/long-way choice. For otherwise identical LBCs, you'll find it easy to exclude either the short-way or long-way solution from further consideration because one these two trajectories will cruise about the Sun in a direction contrary (retrograde) to that of Earth and Mars. Getting your spacecraft onto a retrograde heliocentric trajectory is propellant-prohibitive because the departure planet's speed about the Sun must first be cancelled before additional motion in the opposite direction is imparted. To minimize propellant consumption, your spacecraft trajectory must match departure planet heliocentric velocity at departure (and arrival planet heliocentric velocity at arrival) as closely as possible.

There's actually a further proliferation of Lambert solutions defined by $\mathbf{r}_D$ and $\mathbf{r}_A$ if you're willing to increase Δt to more than an orbit period in the interplanetary cruise ellipse. We'll ignore solutions with $\theta \geq 360°$ in this discussion, as we're not interested in cruising to and from Mars any longer than necessary.

Finally, we should be acutely aware of Lambert solutions with θ near 180° that are marginally short-way or long-way. Corresponding spacecraft trajectories are very nearly Hohmann transfers, sometimes cited as the most propellant-efficient interplanetary trajectories. This would indeed be the case if departure and arrival heliocentric planetary orbits were exactly co-planar. Unfortunately, Mars's orbit plane is tilted (inclined) 1.85° with respect to Earth's (the ecliptic). When $\mathbf{r}_D$ and $\mathbf{r}_A$ are very nearly in opposite directions from the Sun, your spacecraft's trajectory plane is poorly defined. Consequently, Mars's ecliptic inclination can produce Lambert solutions with ecliptic inclinations approaching 90° when θ is near 180°.

Near-Hohmann cruise trajectories are usually extreme "gas-guzzlers" because departing the ecliptic plane always entails a large spacecraft velocity change at Earth and Mars. The only exception would be the very special case when θ is exactly 180° and Mars lies at one of two points (nodes) where its orbit crosses the ecliptic exactly as

[3] If Δt is too small, the long-way solution may require speed sufficient to escape the solar system. Under these conditions, the interplanetary trajectory becomes a hyperbola.

your spacecraft arrives at or departs from Mars. In these unlikely cases, Lambert solutions may still have appreciable ecliptic inclinations depending on how co-linear $\mathbf{r}_D$ and $\mathbf{r}_A$ are interpreted by solution computations. Geometrically speaking, $\theta = 180°$ is a Lambert Theorem singularity leading to an infinite number of equally valid solutions corresponding to all possible heliocentric planes.

With these Lambert Theorem fundamentals understood, you're equipped to design coasted interplanetary cruise trajectories using "minimum impulse" while we await invention of the warp drive. In the following section, you'll learn how to organize and interpret large numbers of Lambert solutions in order to optimize interplanetary trajectories for minimal propellant consumption.

4. The "Pork Chop" Chart (PCC)

A pork chop chart (PCC) is a two-dimensional matrix of values from an ordered array of Lambert solutions. The element value assigned to each solution in the matrix "maps" a third dimension and may be any single variable relatable to all the solutions. In PCC examples to follow, we will confine ourselves to planet-relative departure speed s_D or arrival speed s_A matrix values[4]. Each column in the matrix pertains to a particular Lambert solution departure time t_D, and each row pertains to a specific Lambert solution arrival time t_A.

To better understand the utility of PCCs, let's construct an analogy between them and contour maps. In this concept, t_D corresponds to longitude, t_A corresponds to latitude, and s_D or s_A corresponds to elevation. Indeed, interplanetary mission planners often plot contours from the digital data in PCCs we'll be discussing. Typically, these contours are roughly triangular and reminiscent of a pork chop in shape. In the case of s_D or s_A contours, you're seeking out the "valleys" and other "low spots" for your mission trajectories. Instead of contours, we'll retain digital values in a matrix and color-code them to indicate low (green), intermediate (yellow), or high (red) speeds. When you pick an acceptable value from a PCC, you'll effectively be fixing departure and arrival times for your outbound or return interplanetary cruise trajectory.

By convention, PCCs we'll discuss are rectangular matrices with t_D increasing rightward and t_A increasing downward. Depending on the t_D and t_A values spanned by columns and rows in a particular PCC, some matrix elements may correspond to $\Delta t \le 0$ (a trajectory arriving before or as it departs). Since these elements are nonsensical to our purposes, you'll see them as blanks without any values in the PCC. These void regions in an otherwise rectangular PCC matrix are typically bordered by elements corresponding to very short, but positive, Δt. Since high speed is required to cover interplanetary distances in a short time, elements bordering matrix voids are usually color-coded red.

[4] Because we're neglecting accelerations from Earth and Mars gravity, it's important to remember these planet-relative speeds only apply a couple days after departure from or a couple days before arrival at the planet of interest.

5. Outbound Earth-To-Mars Cruise

Accurately foretelling the pace of technology is doomed to failure, but let's assume we're not ready to begin human exploration of Mars until the year 2030. Figure 2 contains heliocentric plots of Earth and Mars ephemerides projected onto the ecliptic plane during that year. Because Earth's orbit coincides with the ecliptic and Mars's orbit is inclined to the ecliptic by only 1.85°, these projections introduce negligible geometric distortion. Each planet's Figure 2 orbit is annotated with "+" time ticks every 30 days. Ticks are each labeled with the corresponding calendar date in "yyyy-mm-dd" format.

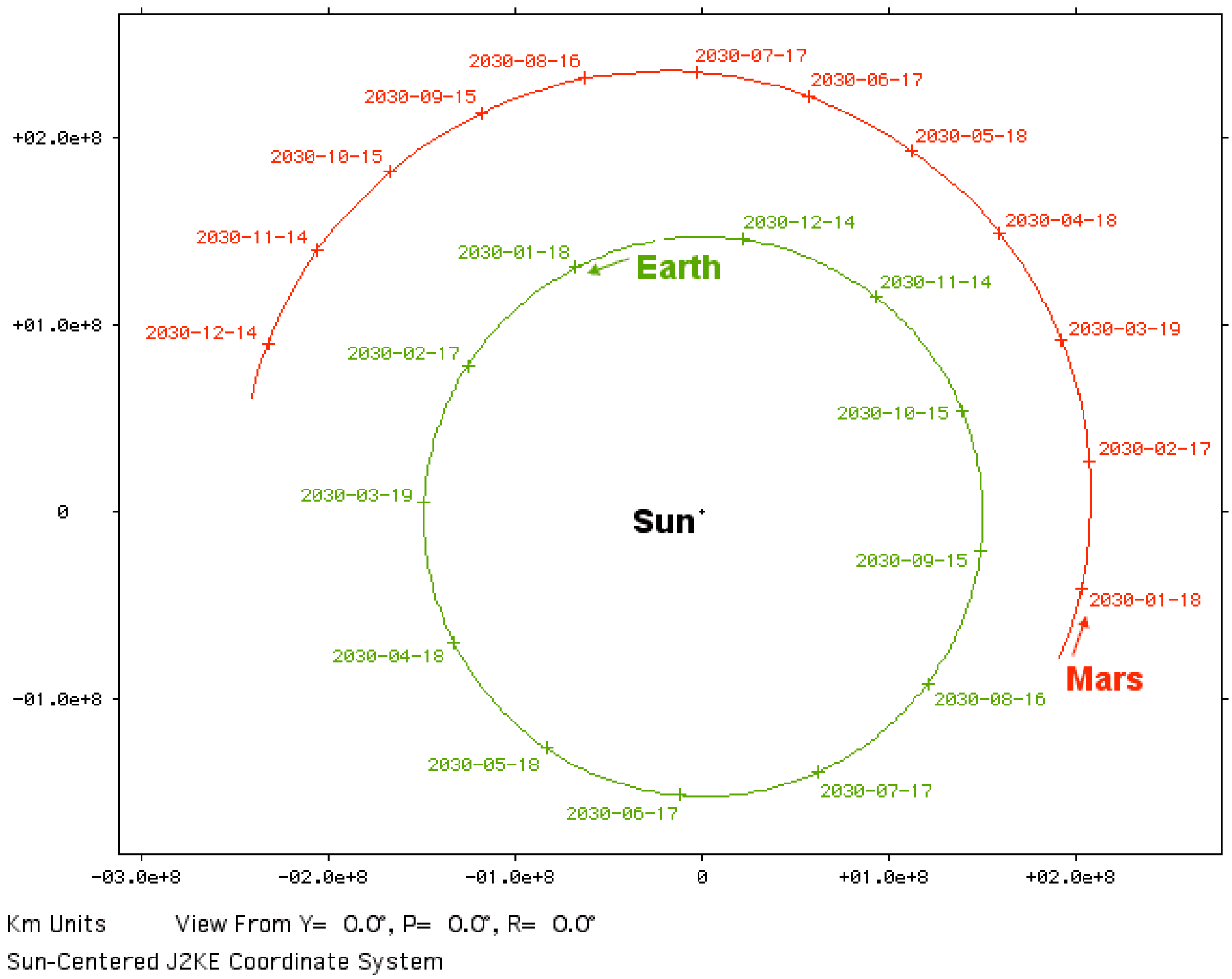

Figure 2: Earth And Mars Heliocentric Motion During The Year 2030

Casual inspection of Figure 2 shows Earth and Mars to be on different sides of the Sun throughout 2030. A closer look indicates the two planets are opposite the Sun (in conjunction) shortly after May 18, 2030. From Figure 2, we can see Earth completes an orbit about the Sun in little more than half the time Mars requires, and it's logical to assume a spacecraft cruising between the two planets would move at an intermediate heliocentrc speed. We therefore need to look for Earth departure times when Mars is about to be "lapped" by Earth. By the end of 2030, Figure 2 indicates this desirable phasing relationship between Earth and Mars will occur in 2031.

Figures 3 and 4 are short-way PCCs respectively containing s_D values departing from Earth and s_A values arriving at Mars in the year 2031. With 14 departure time columns and 14 arrival time rows, speeds from nearly 200 Lambert solutions appear in each PCC. Feel free to zoom in for a closer look at the numbers! Note that dates appear in these PCCs using "mm/dd/yy" format.

Mars Arrive	Earth Depart Date													
Date	1/1/31	1/11/31	1/21/31	1/31/31	2/10/31	2/20/31	3/2/31	3/12/31	3/22/31	4/1/31	4/11/31	4/21/31	5/1/31	5/11/31
5/1/31	10.338	10.494	10.691	11.065	11.732	12.826	14.701	17.714	22.584	30.992	48.419	97.198		
5/11/31	8.808	8.821	8.849	9.011	9.393	10.088	11.368	13.423	16.618	21.679	30.085	47.248	95.192	
5/21/31	7.539	7.453	7.370	7.402	7.621	8.097	9.059	10.614	12.972	16.511	21.803	30.462	47.790	96.433
5/31/31	6.486	6.333	6.180	6.139	6.274	6.644	7.452	8.748	10.664	13.420	17.254	22.894	31.972	49.498
6/10/31	5.619	5.420	5.226	5.153	5.260	5.598	6.348	7.516	9.187	11.497	14.541	18.717	24.740	34.302
6/20/31	4.919	4.686	4.473	4.398	4.514	4.867	5.611	6.716	8.240	10.270	12.833	16.182	20.677	27.055
6/30/31	4.381	4.114	3.894	3.838	3.986	4.378	5.138	6.207	7.630	9.465	11.706	14.531	18.139	22.896
7/10/31	4.033	3.700	3.472	3.443	3.633	4.069	4.846	5.886	7.230	8.917	10.923	13.388	16.425	20.230
7/20/31	3.993	3.458	3.196	3.187	3.413	3.885	4.670	5.681	6.955	8.522	10.347	12.547	15.187	18.375
7/30/31	4.716	3.455	3.070	3.049	3.293	3.783	4.563	5.540	6.750	8.215	9.894	11.890	14.238	16.996
8/9/31	8.811	3.894	3.122	3.017	3.245	3.731	4.493	5.431	6.580	7.956	9.514	11.345	13.469	15.912
8/19/31	55.522	5.543	3.450	3.100	3.255	3.710	4.440	5.335	6.425	7.721	9.175	10.869	12.814	15.016
8/29/31	62.546	14.002	4.371	3.345	3.325	3.710	4.393	5.239	6.271	7.495	8.857	10.436	12.234	14.245
9/8/31	62.689	60.559	7.249	3.909	3.484	3.735	4.348	5.139	6.112	7.268	8.548	10.027	11.701	13.556

Figure 3: Earth Departure Speeds (km/s) PCC

Mars Arrive	Earth Depart Date													
Date	1/1/31	1/11/31	1/21/31	1/31/31	2/10/31	2/20/31	3/2/31	3/12/31	3/22/31	4/1/31	4/11/31	4/21/31	5/1/31	5/11/31
5/1/31	17.373	18.399	19.489	20.669	22.000	23.576	25.574	28.335	32.544	39.834	55.921	102.838		
5/11/31	15.284	16.097	16.940	17.825	18.785	19.873	21.186	22.904	25.356	29.230	36.136	51.649	97.507	
5/21/31	13.476	14.125	14.781	15.450	16.152	16.917	17.802	18.913	20.433	22.712	26.462	33.331	48.858	95.501
5/31/31	11.901	12.419	12.931	13.439	13.955	14.497	15.101	15.837	16.817	18.251	20.521	24.386	31.596	47.347
6/10/31	10.520	10.933	11.332	11.716	12.095	12.478	12.892	13.386	14.036	14.982	16.466	18.908	23.142	31.016
6/20/31	9.310	9.633	9.941	10.229	10.504	10.773	11.055	11.387	11.828	12.480	13.511	15.192	17.995	22.796
6/30/31	8.255	8.496	8.728	8.940	9.136	9.321	9.510	9.734	10.040	10.510	11.270	12.511	14.535	17.831
7/10/31	7.358	7.509	7.672	7.821	7.956	8.079	8.203	8.355	8.576	8.934	9.531	10.509	12.076	14.534
7/20/31	6.667	6.673	6.762	6.856	6.941	7.019	7.099	7.206	7.376	7.669	8.170	8.989	10.277	12.229
7/30/31	6.421	6.015	5.997	6.033	6.076	6.120	6.171	6.253	6.398	6.660	7.109	7.835	8.946	10.575
8/9/31	8.382	5.652	5.397	5.353	5.355	5.372	5.406	5.478	5.617	5.871	6.299	6.971	7.969	9.382
8/19/31	42.599	6.101	5.029	4.831	4.776	4.768	4.793	4.866	5.012	5.273	5.699	6.343	7.264	8.524
8/29/31	48.018	11.649	5.116	4.508	4.352	4.310	4.328	4.408	4.570	4.848	5.280	5.908	6.774	7.920
9/8/31	48.371	46.879	6.690	4.496	4.106	4.002	4.007	4.095	4.274	4.570	5.011	5.626	6.447	7.504

Figure 4: Mars Arrival Speeds (km/s) PCC

Even a quick glance at color-coding in Figures 3 and 4 shows the desirable green regions don't completely overlap. Depending on the propulsion available to your spacecraft at departure and arrival, it may be desirable to minimize one set of speeds at the expense of the other, or strike a compromise between the two. If Earth launch postponements are likely, it may be wise to select the earliest practical departure time.

Values outlined by black boxes in Figures 3 and 4, corresponding to Earth departure on February 20 and Mars arrival on August 19, lead to the example outbound cruise from Earth to Mars plotted in Figure 5. This 6-month example voyage is but one of many possible outbound cruise trajectories. Because the outbound cruise example is inclined to the ecliptic by only 1.89°, Figure 5 plots are all projected onto the ecliptic, as were those in Figure 2.

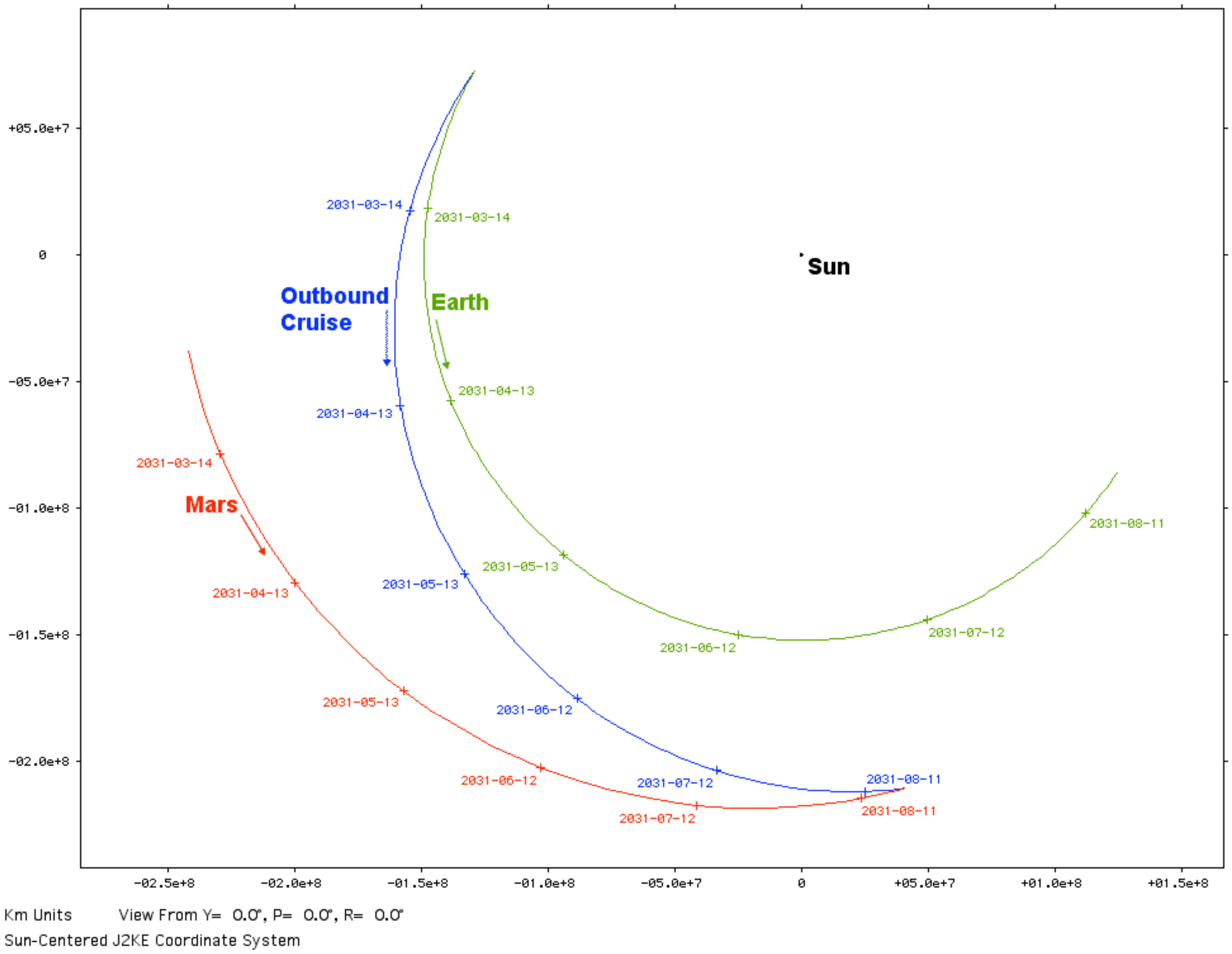

Figure 5: Cruise Departing Earth Feb. 20 and Arriving Mars Aug. 19 In 2031

A small sub-matrix of relatively large speeds reside in the lower left corners of Figures 3 and 4. These PCC elements correspond to near-Hohmann cruises through heliocentric angles slightly less than 180°. Let's consider the 8-month cruise departing from Earth on January 1 and arriving at Mars on September 1 in 2031. This example is plotted in Figure 6. Like Figures 2 and 5, Figure 6 is heliocentric, but its perspective is different. Figures 2 and 5 are viewed from a perspective looking perpendicular to the ecliptic plane because all plots in these illustrations lie within 2° of this plane. Figure 6 is viewed from a perspective 60° away from perpendicular to the ecliptic so we can see the near-Hohmann trajectory's motion out of the ecliptic plane. Distance from the ecliptic plane in Figure 6 appears as dotted lines projected from each time tick. Earth's heliocentric orbit plane coincides with the ecliptic by definition, so no dotted lines are visible in its Figure 6 plot. Short dotted lines are associated with the Mars plot since its ecliptic inclination is 1.85°. In contrast, the near-Hohmann trajectory plot is inclined to the ecliptic by 10.41°, making for relatively prominent dotted projection lines in Figure 6. It's this motion out of the ecliptic plane that contributes relatively large speeds with respect to Earth and Mars in the near-Hohmann regions of Figures 3 and 4. These are obviously PCC regions to avoid in your outbound mission planning!

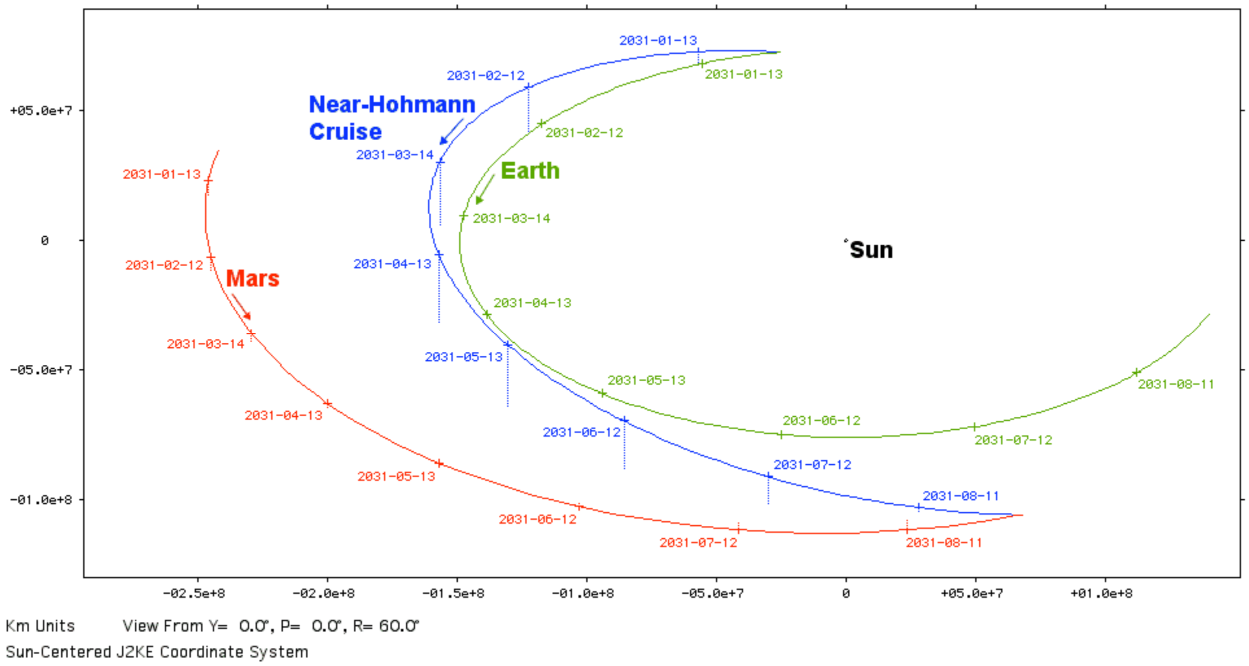

Figure 6: Cruise Departing Earth Jan. 1 and Arriving Mars Sep. 1 In 2031

6. Return Mars-To-Earth Cruise

Since we're confined to cruise intervals between Earth and Mars lasting 6 months or more, Figure 5 tells us a sobering story about our return trip. By the time we reach Mars in late 2031, Earth has already phased ahead of us in orbit around the Sun to the point a cruise lasting 6 months or so can't catch up without taking a "short-cut" inside Earth's orbit. As you can imagine, both s_D from Mars and s_A at Earth for these short-cut return trajectories would be color-coded red on a PCC.

Under interplanetary cruise constraints we've previously adopted, our only remedy is to wait at Mars until Earth begins to close in from behind again. The "cycle-time" (synodic period) required for Earth to phase completely around the Sun with respect to the Mars-Sun line is 780 days, or a bit less than 26 months. That means our return cruise short-way PCCs, as shown in Figures 7 and 8, need to span dates in the year 2033.

Earth Arrive	Mars Depart Date												
Date	1/1/33	1/11/33	1/21/33	1/31/33	2/10/33	2/20/33	3/2/33	3/12/33	3/22/33	4/1/33	4/11/33	4/21/33	5/1/33	5/11/33
7/1/33	4.249	4.300	4.373	4.478	4.627	4.833	5.118	5.505	6.029	6.733	7.678	8.954	10.701	13.170
7/11/33	3.648	3.670	3.713	3.784	3.894	4.055	4.284	4.602	5.033	5.610	6.375	7.386	8.728	10.542
7/21/33	3.184	3.186	3.209	3.260	3.348	3.485	3.685	3.966	4.347	4.854	5.515	6.372	7.480	8.927
7/31/33	2.846	2.834	2.843	2.883	2.960	3.086	3.274	3.538	3.894	4.361	4.960	5.723	6.687	7.913
8/10/33	2.625	2.599	2.598	2.629	2.700	2.822	3.004	3.260	3.601	4.044	4.603	5.302	6.170	7.249
8/20/33	2.516	2.469	2.453	2.474	2.539	2.655	2.833	3.082	3.411	3.834	4.362	5.013	5.810	6.785
8/30/33	2.534	2.444	2.400	2.401	2.452	2.558	2.726	2.964	3.281	3.684	4.183	4.795	5.534	6.428
9/9/33	2.762	2.557	2.446	2.404	2.425	2.509	2.659	2.880	3.178	3.559	4.031	4.605	5.295	6.122
9/19/33	3.721	2.996	2.662	2.507	2.462	2.502	2.620	2.815	3.088	3.443	3.886	4.424	5.069	5.837
9/29/33	24.409	5.749	3.545	2.872	2.620	2.557	2.613	2.765	3.004	3.328	3.738	4.239	4.842	5.557
10/9/33	42.419	42.426	34.596	5.514	3.304	2.785	2.673	2.742	2.928	3.211	3.583	4.047	4.607	5.272
10/19/33	42.377	42.587	42.788	42.940	41.017	4.882	3.046	2.798	2.874	3.097	3.426	3.849	4.366	4.983
10/29/33	42.205	42.461	42.718	42.976	43.230	43.470	43.297	3.828	2.914	2.998	3.268	3.647	4.121	4.692
11/8/33	41.945	42.242	42.541	42.842	43.143	43.445	43.744	44.041	44.308	2.951	3.106	3.445	3.878	4.402

Figure 7: Mars Departure Speeds (km/s) PCC

Earth Arrive Date	Mars Depart Date 1/1/33	1/11/33	1/21/33	1/31/33	2/10/33	2/20/33	3/2/33	3/12/33	3/22/33	4/1/33	4/11/33	4/21/33	5/1/33	5/11/33
7/1/33	11.175	11.436	11.708	11.994	12.298	12.627	12.989	13.396	13.869	14.436	15.146	16.074	17.351	19.220
7/11/33	9.536	9.742	9.954	10.174	10.405	10.650	10.915	11.209	11.543	11.937	12.420	13.040	13.873	15.054
7/21/33	8.086	8.245	8.408	8.576	8.749	8.931	9.124	9.335	9.572	9.847	10.181	10.605	11.170	11.960
7/31/33	6.790	6.908	7.029	7.153	7.280	7.412	7.552	7.702	7.870	8.064	8.300	8.600	9.001	9.561
8/10/33	5.727	5.806	5.889	5.975	6.065	6.159	6.258	6.365	6.486	6.627	6.800	7.025	7.329	7.756
8/20/33	4.866	4.900	4.944	4.997	5.055	5.118	5.188	5.266	5.356	5.465	5.603	5.786	6.038	6.393
8/30/33	4.275	4.238	$\boxed{4.232}$	4.244	4.271	4.308	4.355	4.414	4.487	4.581	4.705	4.872	5.104	5.429
9/9/33	4.156	3.956	3.856	3.809	3.795	3.804	3.832	3.877	3.942	4.032	4.154	4.322	4.551	4.867
9/19/33	5.175	4.240	3.846	3.661	3.575	3.543	3.546	3.578	3.638	3.727	3.852	4.024	4.257	4.568
9/29/33	35.542	7.941	4.880	4.029	3.712	3.584	3.543	3.554	3.603	3.690	3.816	3.989	4.220	4.524
10/9/33	62.832	62.569	50.393	7.548	4.596	3.967	3.785	3.745	3.773	3.849	3.968	4.134	4.355	4.643
10/19/33	62.932	62.997	63.016	62.916	59.557	6.652	4.368	4.061	4.030	4.084	4.190	4.343	4.548	4.814
10/29/33	62.797	62.929	63.033	63.107	63.145	63.126	62.390	5.398	4.432	4.424	4.513	4.649	4.833	5.072
11/8/33	62.510	62.700	62.862	62.997	63.105	63.183	63.232	63.247	63.174	4.791	4.872	4.993	5.148	5.352

Figure 8: Earth Arrival Speeds (km/s) PCC

Slightly favoring minimal Mars departure speed over that at Earth arrival, PCC elements outlined by black boxes in Figures 7 and 8 indicate a return cruise departing from Mars January 21 and arriving at Earth on August 30 has been selected as an example for plotting in Figure 9. You might select a similar return option if your spacecraft will use acceleration from Earth atmospheric friction to slow its return speed. Should Earth return require propulsive braking, however, a delayed Mars departure and delayed Earth return may be advisable. Because the return cruise has ecliptic inclination of only 1.66°, Figure 9 perspective is again normal to the ecliptic as in Figures 2 and 5.

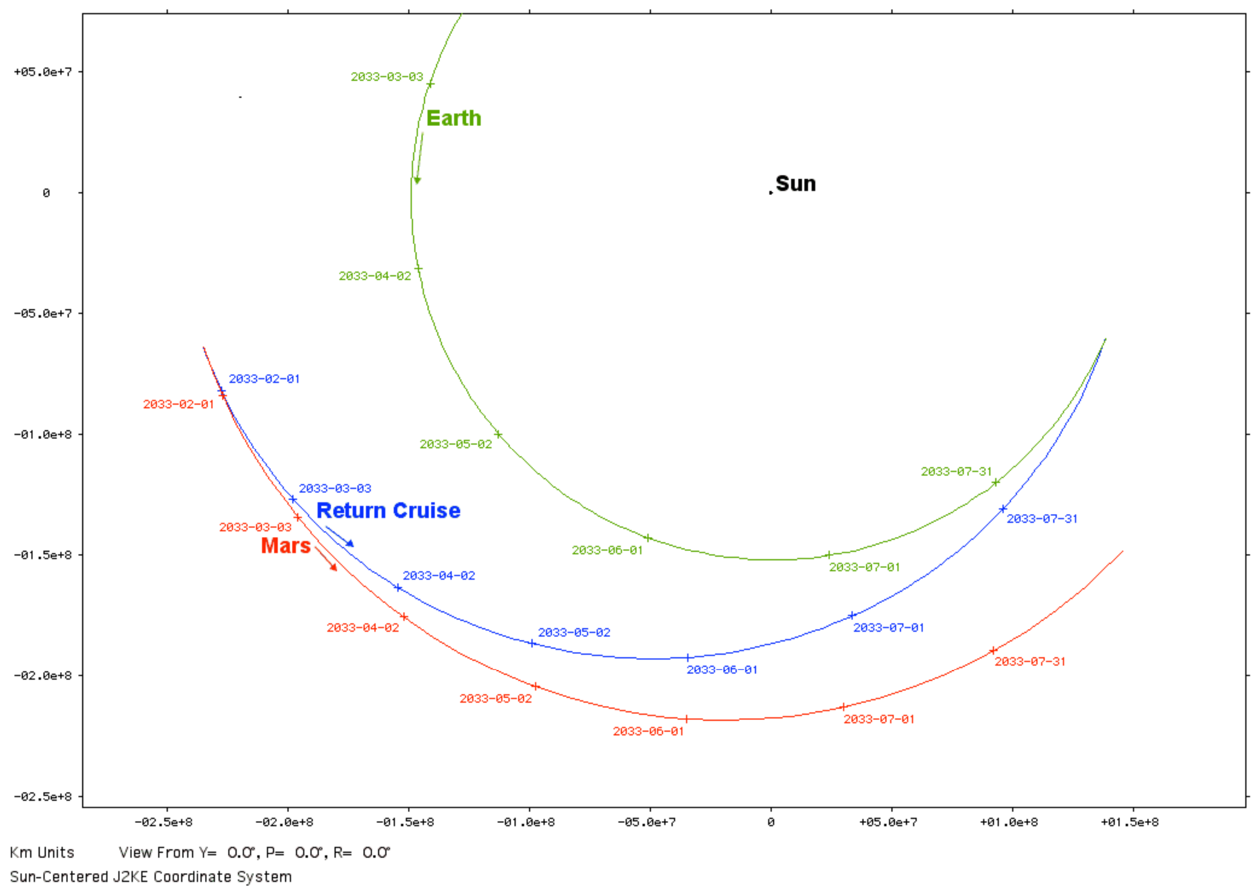

Figure 9: Cruise Departing Mars Jan. 21 and Arriving Earth Aug. 30 In 2033

7. Conclusion

Welcome back to Earth! Our example round trip requires an outbound cruise to Mars lasting 6 months, a loiter period at Mars lasting 17 months, and a return to Earth cruise lasting 7 months. We'll be away from home a total of 30 months, or 2.5 years. Although most of our journey is presumably spent exploring Mars, using that warp drive for propulsion is starting to sound better and better!

Even foreseeable advances in propulsion, such as Ad Astra Rocket Company's Variable Specific Impulse Magnetoplasma Rocket (VASIMR™), can make a big difference in your round trip timeline. Using high-powered VASIMR engines, your spacecraft could reach Mars in less than 7 weeks. That would permit more than a month of Mars exploration before Earth started phasing ahead of Mars, triggering an equally rapid return. Such a round trip could last less than 6 months.

Hopefully, you now have a more complete appreciation of the trade-off considerations, techniques, and terminology involved in planning trajectories between Earth and Mars. Other than specific numbers, all the *Interplanetary Cruising* material you've studied is applicable to any planet you might want to visit in our solar system. Enjoy the ride!